substituted into (103) with the force law of (104), we obtain

$$(106)\quad \mathrm{d}^2S/\mathrm{d}y^2 = [S(y-\theta) - 2S(y) + S(y+\theta)]\cdot\{1-\alpha[S(y-\theta) - S(y+\theta)]\}\,,$$

where

$$y = T - N\theta\,.$$

Equation (106) is an uncommon type of differential-difference equation and is not readily solved, even numerically. An approximate solution can be found by expanding $S(y\pm\theta)$ in powers of θ and retaining fourth order terms. If dissipation is nonzero, however small, nearly-steady oscillations of the type suggested by Fig. 32 are obtained. The frequencies of oscillation predicted by the approximate theory are, however, quite different from those obtained from numerical integration of the transient equations. It thus appears that this picture of the permanent regime in the lattice is qualitatively correct, but that eq. (106) must be solved if parameters of the permanent regime are to be calculated [21].

One interesting application of these results is to the von Neumann-Richtmyer integration of the flow equations. If artificial viscosity is too small, the results of such an integration oscillate wildly. This has sometimes been interpreted as instability of the numerical integration procedure; it is in fact the true physical behavior of the lumped-constant system used to model the continuum for purposes of numerical integration.

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